Further Results on Stolarsky-3 Mean Graphs

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ABSTRACT

Let G = (V, E) be a simple graph. G is said to be Stolarsky-3 Mean graph if each vertex $x \in V$ is assigned distinct labels f(x) from 1,2,….,q+1 and each edge e = uv is assigned with the labels f(e) = $\left\lceil \sqrt{\frac{[(f(u))^2+f(u)f(v)+(f(v))^2]}{3}} \right\rceil$ **(or)** $\left\lfloor \sqrt{\frac{[(f(u))^2+f(u)f(v)+(f(v))^2]}{3}} \right\rfloor$ *then the resulting edge labels are distinct and f is called a Stolarsky-3 Mean labeling of G. In this paper, we show the graphs $C_n @ P_m$, $L_n \Theta \overline{K_2}$, $TL_n \Theta K_1$, $(C_m \Theta K_3) \cup L_n$, $C_m \cup T_n$ etc. are Stolarsky-3 mean graphs.*

Keywords: *Graph Labeling, Stolarsky-3 mean labeling, Dragon graph, Comb graph, Ladder graph, Triangular ladder graph and Triangular snake graph.*

1. Introduction

Let G be a finite, undirected and simple graph with p vertices and q edges. There are several types of labeling and a detailed survey was done by [1]. The standard terminology and notations in this article are based on the book Graph theory [2]. The concept of Mean labeling was introduced by Somasundaram et.al. in [3]. Motivated by the concept of [3], Kavitha et.al [4] introduced a new concept namely Stolarsky-3 mean graph and proved that the path graph, cycle graph, comb graph, ladder graph, star graph, triangular snake graph and quadrilateral graph are Stolarsky-3 mean graphs. Sandhya et.al. [5] proved that slanting Ladder, triangular ladder, H-graph, twig graph, middle graph and total graph are Stolarsky-3 mean graphs.

We provide the following definitions which are necessary for our main results.

Definition 1.1: A graph G with p vertices and q edges is said to be Stolarsky-3 Mean graph if each vertex $x \in V$ is assigned distinct labels f(x) from 1,2, ... ,q+1 and each edge e = uv is assigned with the labels f(e) = $\left\lceil \sqrt{\frac{[(f(u))^2+f(u)f(v)+(f(v))^2]}{3}} \right\rceil$ **(or)** $\left\lfloor \sqrt{\frac{[(f(u))^2+f(u)f(v)+(f(v))^2]}{3}} \right\rfloor$ then the resulting edge labels are distinct and f is called a Stolarsky-3 Mean labeling of G.

Definition 1.2. A walk in which all the vertices $u_1, u_2, \dots, u_n$ are distinct is called a path. It is denoted by P_n.

Definition 1.3: A closed path is called a cycle. A cycle on n vertices is denoted by C_n.

Definition 1.4. The Corona $G_1 \Theta G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has P_1 vertices) and P_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2.

Definition 1.5. The Cartesian product $G_1 \times G_2$ of two graphs is defined to be the graph with vertex set $V_1 \times V_2$ and two vertices U = (U_1, U_2) and V = (V_1, V_2) are adjacent in $G_1 \times G_2$ if either $U_1 = V_1$ and U_2 is adjacent to V_2 and U_1 is adjacent to V_1.

Definition 1.6. The Union $G_1 \cup G_2$ of two graphs G_1 and G_2 is the graph with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$. The union of m copies of G is denoted by mG.

Definition 1.7. A Dragon is a graph obtained by joining an end vertex of a path P_m to a vertex of the cycle C_n. It is denoted by $C_n @ P_m$.

Definition 1.8. Comb $P_n \Theta K_1$ is a graph obtained by joining a single pendant edge to each vertex of a path.

Definition 1.9. The Ladder graph L_n $(n \geq 2)$ is the product graph $P_2 \times P_n$ which contains 2n vertices and 3n-2 edges.

Definition 1.10. A Triangular ladder TL_n is a graph obtained from L_n by adding the edges $u_i v_{i+1}, 1 \leq i \leq n-1$, where u_i and v_i $1 \leq i \leq n$ are the vertices of L_n such that $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ are two paths of length n in the graph L_n.

Definition 1.11. A Triangular Snake T_n is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \leq i \leq n-1$. That is, every edge of a path is replaced by a triangle C_3.

Definition 1.12. The graphs $\overline{K_p}$ are totally disconnected and are regular of degree 0.

2 Main Results

Theorem 2.1. Let P_n be the path and G be the graph obtained from P_n by attaching a pendant edge to both sides of each vertex of P_n. Then G is a Stolarsky-3 mean graph.

Proof: Let $u_1, u_2, \ldots, u_n$ be the vertices of the path P_n. Let G be a graph obtained from P_n by attaching a pendant edge to both sides of each vertex of P_n. Let v_i, w_i, $1 \leq i \leq n$ be the new vertices of G.

Define a function $f: V(G) \rightarrow \{1, 2, \ldots, q+1\}$ by

$f(u_i) = 3i-1, 1 \leq i \leq n.$

$f(v_i) = 3i-2, 1 \leq i \leq n.$

$f(w_i) = 3i, 1 \leq i \leq n.$

Edge labeling's are

$f(u_i u_{i+1}) = 3i$, $1 \le i \le n-1$.

$f(u_i v_i) = 3i - 2$, $1 \le i \le n$.

$f(u_i w_i) = 3i - 1$, $1 \le i \le n$. Here the edge labels are distinct. Hence G is Stolarsky-3 mean graph.

Example 2.2. The graph G obtained from P_5 is given below.

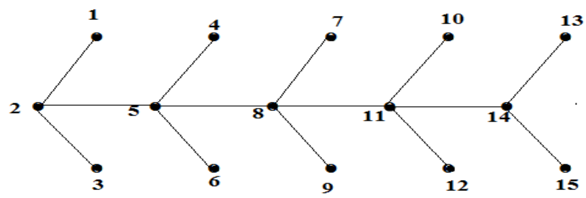

Figure 1

Theorem 2. 3. The graph obtained by attaching $K_{1,2}$ to each pendant vertex of a comb $P_n \boldsymbol{\Theta} K_1$ forms a Stolarsky-3 mean graph.

Proof: Let G be a graph obtained by attaching $K_{1,2}$ to each pendant vertex of a comb. Let u_i , $\boldsymbol{v_i}$, x_i, y_i, $1 \le i \le n$ be the vertices of G.

Define a function f: V(G)→ {1,2, 3,…,q+1} by

$f(u_i) = 4i-3$, $1 \le i \le n$.

$f(v_i) = 4i-2$, $1 \le i \le n$.

$f(x_i) = 4i-1$, $1 \le i \le n$.

$f(y_i) = 4i$, $1 \le i \le n$.

The edges are labeled as

$f(u_i u_{i+1}) = 4i$, $1 \le i \le n-1$.

$f(u_i v_i) = 4i-3$, $1 \le i \le n$.

$f(x_i v_i) = 4i-2$, $1 \le i \le n$.

$f(x_i v_i) = 4i-1$, $1 \le i \le n$.

Then we get distinct edge labels. Hence G is Stolarsky-3 mean graph.

Example 2.4. The Stolarsky-3 mean labeling of $(P_4 \boldsymbol{\Theta} K_1) \boldsymbol{\Theta} K_{1,2}$

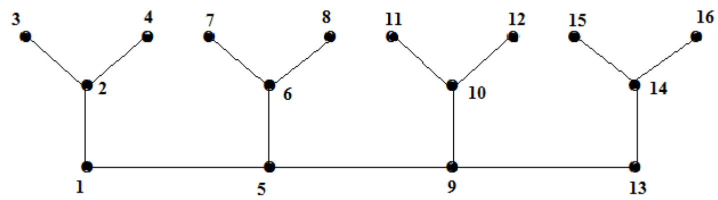

Figure 2

Theorem 2.5. The Dragon graph $C_n@ P_m$ is Stolarsky-3 mean graph.

Proof: Let $u_1, u_2, \dots, u_n$ be the vertices of the cycle C_n and $v_1, v_2, \dots, v_m$ be the vertices of the path P_m.

Here $u_n = v_1$. Define the function f: V ($C_n@ P_m$)→{ 1,2,…., q+1} by

$f(u_i)$ = i, $1 \le i \le n$.

$f(v_1) = f(u_n)$.

$f(v_i)$ = (n-1) + i, $2 \le i \le m$. Then the edge labels are distinct.

Hence the Dragon graph $C_n@ P_m$ is Stolarsky-3 mean graph.

Example 2.6. The Stolarsky-3 mean labeling pattern of $C_5@ P_5$ is shown below.

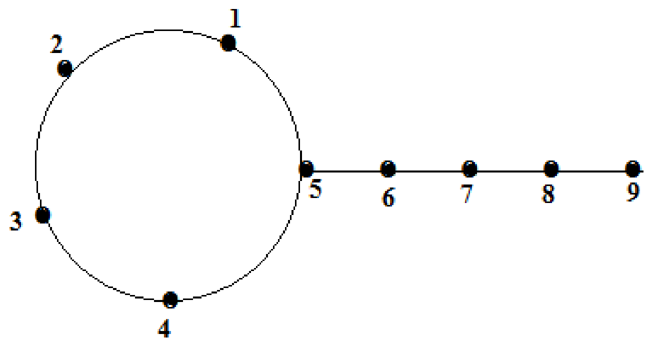

Figure 3

Theorem 2.7. Let G be the graph obtained from a path P_n by attaching C_3 in both end edges of P_n. Then G is a Stolarsky-3 mean graph.

Proof: Let $u_1, u_2, \dots, u_n$ be the vertices of the path P_n and $u_1u_2v_1$, $u_{n-1}u_nv_2$ be the triangles which are attached to the path at both ends.

Define a function f: V(G)→ {1,2,3,…,q+1} by

$f(u_1)$ = 1.

$f(u_i)$ = (i-1)+2, $2 \le i \le n$-1.

$f(u_n)$ = n+3.

$f(v_1)$ = 2.

$f(v_2)$ = n+2.

Then the edges are labeled as

$f(u_1v_1)$ =1.

$f(u_2v_1)$ =3.

$f(u_1u_2)$ =2.

f(u_iu_{i+1}) =i+2, 2≤ $i \leq n-2$.

f($u_{n-1}u_n$) =n+2.

f($u_{n-1}v_2$) =n+1.

f(u_nv_2) =n+3. Then the edge labels are distinct.

Hence G is Stolarsky-3 mean graph.

Example 2.8. The Stolarsky-3 mean labeling of G obtained from P_7 is given below.

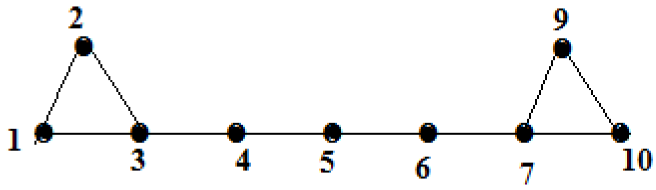

Figure 4

Theorem 2.9. The graph obtained by attaching K_3 to each pendant vertex of a comb $P_n \Theta K_1$ forms a Stolarsky-3 mean graph.

Proof: Let G be a graph obtained by attaching K_3 to each pendant vertex of a comb. Let u_i , v_i, x_i, y_i, 1≤ i ≤ n be the vertices of G.

Define a function f : V(G)→ {1,2,3,…, q+1} by

f(u_i) = 5i-3, 1≤ $i \leq n$.

f(v_i) = 5i-4, 1≤ $i \leq n$.

f(x_i) = 5i-1, 1≤ $i \leq n$.

f(y_i) = 5i, 1≤ $i \leq n$.

The edges are labeled as

f(u_iu_{i+1}) = 5i, 1≤ i ≤ n-1.

f(u_iv_i) = 5i-4, 1≤ $i \leq n$.

f(x_iv_i) = 5i-3, 1≤ $i \leq n$.

f(y_iv_i) = 5i-2, 1≤ $i \leq n$

f(x_iy_i) = 5i-1.

Then the edge labels are distinct. Hence G is Stolarsky-3 mean graph.

Example 2.10. The Solarsky-3 mean labeling of $(P_4 \Theta K_{1,2}) \Theta K_3$

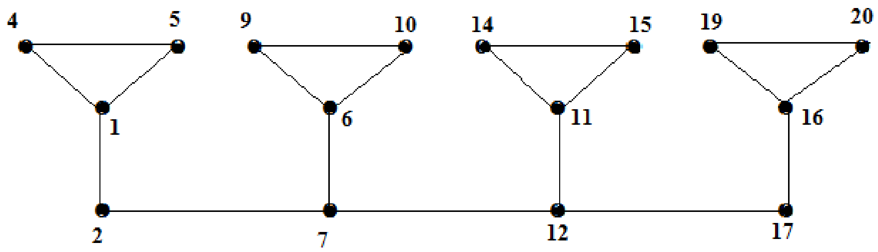

Figure 5

Theorem 2.11. The graph $L_n \, \Theta \, \overline{K_2}$ is a Stolarsky-3 mean graph.

Proof: Let L_n be the Ladder graph with the vertices u_i, v_i, $1 \leq i \leq n$.

The graph G = $L_n \, \Theta \, \overline{K_2}$ is obtained by joining u_i with two new vertices x_i, y_i, $1 \leq i \leq n$ and join v_i, with two new vertices s_i, t_i, $1 \leq i \leq n$ in L_n

Define a function f: V(G)→ {1,2,3,…,q+1} by

$f(u_i) = 7i-4, 1 \leq i \leq n.$

$f(v_i) = 7i-5, 1 \leq i \leq n.$

$f(x_i) = 7i-3, 1 \leq i \leq n.$

$f(y_i) = 7i-1, 1 \leq i \leq n.$

$f(s_i) = 7i-6, 1 \leq i \leq n.$

$f(t_i) = 7i-2, 1 \leq i \leq n.$

Then the edges are labeled as

$f(u_i u_{i+1}) = 7i, 1 \leq i \leq n-1.$

$f(u_i v_i) = 7i-5, 1 \leq i \leq n.$

$f(v_i v_{i+1}) = 7i, 1 \leq i \leq n-1.$

$f(u_i x_i) = 7i-4, 1 \leq i \leq n.$

$f(u_i y_i) = 7i-2, 1 \leq i \leq n.$

$f(v_i s_i) = 7i-6, 1 \leq i \leq n.$

$f(v_i t_i) = 7i-3, 1 \leq i \leq n.$ Thus, we get distinct edge labels.

Hence G is Stolarsky-3 mean graph.

Example 2.12. The Stolarsky-3 mean labeling of L_4 $\boldsymbol{\Theta}$ $\overline{K_2}$

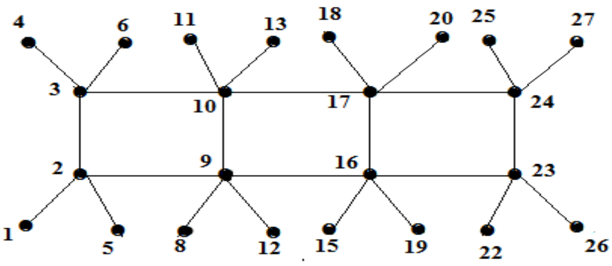

Figure 6

Theorem 2.13. TL_n Θ K_1 is Stolarsky-3 mean graph.

Proof: Let TL_n be the Triangular ladder graph with the vertices u_i, v_i, 1≤ $i \leq n$.

Let G= TL_n Θ K_1 be a graph obtained by attaching x_i to u_i and y_i to v_i in TL_n.

Define a function f: V(G)→ {1,2,3,...,q+1} by

$f(u_i)$ = 6i-5, 1≤ $i \leq n$.

$f(v_i)$ = 6i-3, 1≤ $i \leq n$.

$f(x_i)$ = 6i-4, 1≤ $i \leq n$.

$f(y_i)$ = 6i-2, 1≤ $i \leq n$.

Then the edge labels are

f($u_i u_{i+1}$)= 6i, -2, 1≤ i ≤ n-1.

$f(v_i v_{i+1})$ = 6i, 1≤ i ≤ n-1.

$f(u_i x_i)$ = 6i-5, 1≤ $i \leq n$, $f(u_i v_{i+1}$)= 6i-1, 1≤ $i \leq n$.

f ($v_i y_i$) =6i-3, 1≤ $i \leq n$. Then the edge labels are distinct.

Hence G is Stolarsky-3 mean graph

Example 2.14. The Stolarsky-3 mean labeling of TL_5 Θ K_1

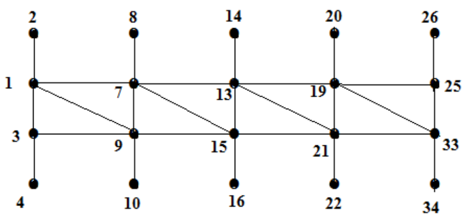

Figure 7

Theorem 2.15. (C_m Θ K_3) ∪ L_n is Stolarsky-3 mean graph.

Proof: Let $u_1, u_2, \ldots, u_m$, u_1 be the vertices of the cycle C_m and let K_3 be the cycle with the vertices $v_1, v_2, \ldots, v_m$, $w_1, w_2, \ldots, w_m$ which are attached to the vertices of the cycle C_m.

Let L_n be the Ladder graph with the vertices x_i and y_i, 1≤ i ≤ n. Let G = (C_m Θ K_3) ∪ L_n.

Define a function f: V(G) →{1,2,…., q+1} by

f(u_i) = 4i-2, 1≤ $i \leq m$.

f(v_i) = 4i -3, 1≤ $i \leq m$.

f(w_i) = 4i , 1≤ $i \leq m$.

f(x_i) =4m+(3i-2) , 1≤ $i \leq n$.

f(y_i) =4m+(3i-1), 1≤ $i \leq n$. Then we obtain distinct edge labels.

Hence (C_m ʘ $K_{1,2}$) ∪ L_n is Stolarsky-3 mean graph.

Example 2.16. The Stolarsky-3 mean labeling of (C_5 ʘ K_3) ∪ L_5 is given below

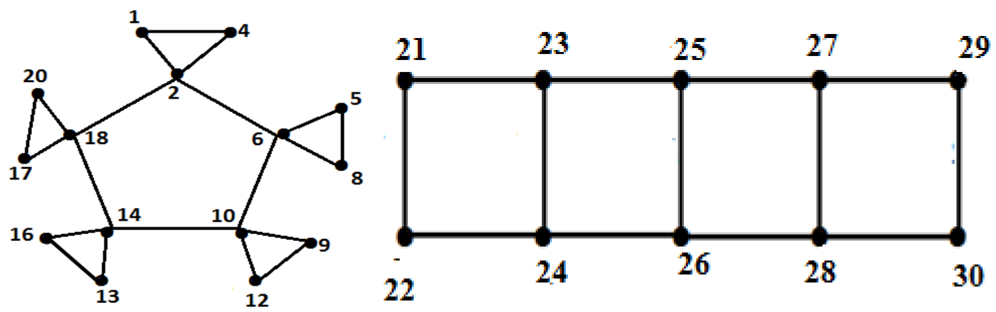

Figure 8

Theorem 2.17. C_m ∪ T_n is Stolarsky-3 mean graph.

Proof: Let $u_1, u_2, \dots, u_m, u_1$ be the vertices of the cycle C_m and let T_n be the Triangular snake graph with the vertices $v_1, v_2, \dots, v_n$ and $w_1, w_2, \dots, w_{n-1}$. Let G = C_m ∪ T_n. Define a function f: V(G) →{1,2,…, q+1} by

f(u_i) =i, 1≤ $i \leq m$.

f(v_i) = m+(3i-2), 1≤ $i \leq n$.

f(w_i) = m+(3i-1), 1≤ $i \leq n-1$. Then we get distinct edge labels.

Hence C_m ∪ T_n is Stolarsky-3 mean graph.

Example 2.18. The Stolarsky-3 mean labeling of C_5 ∪ T_5 is given below

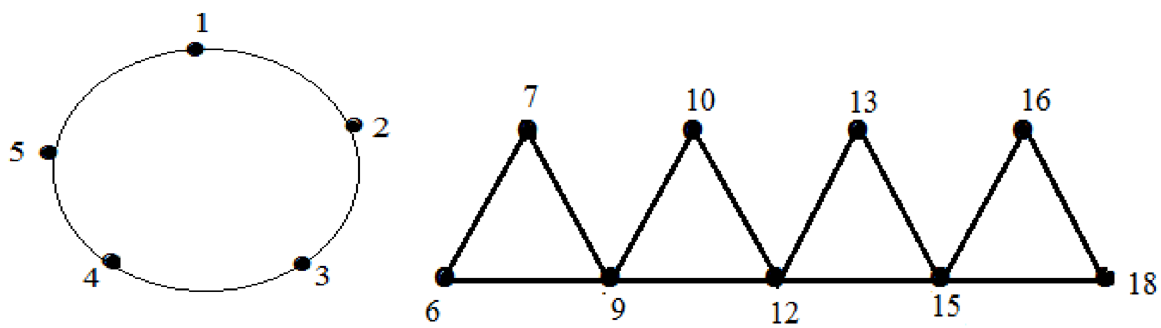

Figure 9

Conclusion

The Study of labeled graph is important due to its diversified applications. It is very interesting to investigate Stolarsky-3 mean labeling of some new graphs. The derived results are demonstrated by means of sufficient illustrations which provide better understanding. It is possible to investigate similar results for several other graphs.

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